LITERATURE CITED

1. P. F. Zil'berman, P. A. Savintsev, and Zh. A. Isakov, Fiz. Khim. Obrab. Mater., No. 5, 86-88 (1981).
2. P. F. Zil'berman, P. A. Savintsev, and Zh. A. Isakov, Neorg. Mater., No. 11, 2080-2083 (1981).
3. P. A. Savintsev, P. F. Zil'berman, and Zh. A. Isakov, Zh. Fiz. Khim., 55, No. 3, 783-784 (1981).
4. V. I. Rogov and P. A. Savintsev, Contact Melting of Metals [in Russian], Nal'chik (1983).
5. P. A. Savintsev and V. I. Rogov, Zavod. Lab., No. 2, 195 (1969).
6. S. De Groot and O. Mazur, Nonequilibrium Thermodynamics [in Russian], Moscow (1962).
7. P. F. Zil'berman, Izv. Vyssh. Uchebn. Zaved., Fiz., No. 4, 128 (1981).
8. P. F. Zil'berman, P. A. Savintsev, A. S. Ksenofontov, and R. M. Gazaryan, Inzh.-Fiz. Zh., 47, No. 5, 771-773 (1984).
9. A. A. Samarskii, Theory of Difference Schemes [in Russian], Moscow (1977).

STEFAN PROBLEM IN THE THEORETICAL MODEL OF THE THERMAL INTERACTION BETWEEN A MOLTEN HEAT-LIBERATING MATERIAL AND FINITE WALLS

G. N. Vlasichev, G. B. Usynin, and Yu. I. Anoshkin

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An approximate analytical solution of the one-dimensional Stefan problem is obtained for a single finite wall with constant heat fluxes at the boundaries. The conjugate problem for a system of molten heat-liberating material with two walls is solved by the finite-difference method.

In investigating the safe operating conditions of fast reactors, it is necessary to analyze the thermal interaction between the molten heat-liberating fuel and the casing walls of the malfunctioning and neighboring heat-liberating piles. Since this interaction is accompanied by melting of the casing wall and motion of its front, the analysis involves solving the problem of phase transition, called the Stefan problem. Accurate analytical solutions of the phase-transition problem are only known for individual cases of a semiinfinite media [1, 2]. An approximate analytical solution of the one-dimensional Stefan problem may be obtained for a single wall of finite thickness with constant heat fluxes at the boundaries ($q_{s1} > q_{s2}$). The solution is found by an integral balance method using the Lebenson method.

Integrating the one-dimensional heat-conduction equation

$$c(x,\ t)\frac{\partial T(x,\ t)}{\partial t}-\frac{\partial}{\partial x}\lambda(x,\ t)\frac{\partial T(x,\ t)}{\partial x} \tag{1}$$

within the limits of the liquid and solid phases of the wall, and using the Stefan condition

$$-\lambda_l\frac{\partial T}{\partial x}\Big|_{x=y(t)-0}+\lambda_s\frac{\partial T}{\partial x}\Big|_{x=y(t)+0}=R_m\frac{\partial y(t)}{\partial t}, \tag{2}$$

it is found that

$$R_m\frac{\partial y(t)}{\partial t}=q_{s1}-q_{s2}-c_l\int\limits_{X_1(t)}^{y(t)}\frac{\partial T(x,\ t)}{\partial t}\,\partial x-c_s\int\limits_{y(t)}^{\delta}\frac{\partial T(x,\ t)}{\partial t}\,\partial x. \tag{3}$$

Taking account of the downward runoff of the molten wall material under its own weight, it is assumed that the coordinate of the molten-layer boundary is determined by the expression $X_1(t) = by(t)$, where b is some constant ($0 \leq b \leq 1$).

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TABLE 1. Comparison of the Results of Numerical and Accurate Analytical Solution for a Single Wall with a Constant Flux at One Boundary and a Zero Flux at the Other

y/δ	0,42	0,83	0,93	1
t, sec (numerical solution)	2,55	5,05	5,65	—
t, sec (from Eq. (5))	2,56	5,04	5,62	6,05

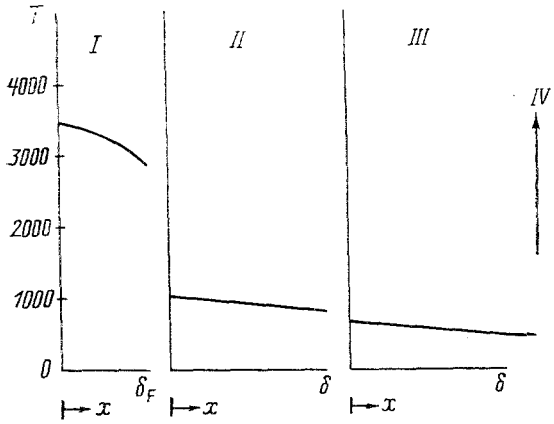

Fig. 1. Diagram and initial temperature state of the system before contact of molten fuel with the casing wall of the malfunctioning fuel assembly: I) fuel; II) casing wall of the malfunctioning fuel assembly; III) casing wall of the adjacent fuel assembly; IV) heat carrier of the adjacent fuel assembly, T, °C.

By analogy with the first Lebenson method, in which the temperature distribution of the corresponding steady state is taken [1], in the present case the temperature distribution is written in the form

$$T(x,\ t)=\begin{cases}T_m+(q_{s1}/\lambda_l)\,[y(t)-x],\ X_1(t)\leqslant x\leqslant y(t),\\ T_m-(q_{s2}/\lambda_s)\,[x-y(t)],\ y(t)\leqslant x\leqslant\delta.\end{cases} \tag{4}$$

Setting y(0) = 0, the following relation is obtained between the phase-boundary position y(t) and the time t

$$t=\frac{1}{1-q_{s2}/q_{s1}}\left\{\frac{R_m}{q_{s1}}\,y(t)+\frac{(1-b)\,y^2(t)}{2a_l}+\frac{q_{s2}}{q_{s1}}\,\frac{[2\delta-y(t)]\,y(t)}{2a_s}\right\}. \tag{5}$$

From Eq. (5), the time for the whole wall to melt is

$$t_\delta=\frac{1}{1-q_{s2}/q_{s1}}\left\{\frac{R_m}{q_{s1}}\,\delta+\left[\frac{1-b}{2a_l}+\frac{1}{2a_s}\,\frac{q_{s2}}{q_{s1}}\right]\delta^2\right\}. \tag{6}$$

With conservation of the molten layer (b = 0) and/or nonzero heat transfer from the external boundary ($0 < q_{S2} < q_{S1}$), the melting time t_δ must increase considerably with increase in wall thickness; the thermal diffusivity of the liquid and solid phases of the wall material is significant here. With a small wall thickness or total removal of the molten material (b = 1), the melting time is determined basically by the latent heat of fusion and the heat-flux values at the boundaries.

In the general case of interaction between molten fuel and the casing wall of a malfunctioning fuel-assembly surrounded by fuel assemblies with normal heat-extraction conditions, a conjugate heat-transfer problem must be solved. Suppose that, in a system consisting of a layer of molten heat-liberating fuel and two heat-transmitting walls of the fuel-assembly casing separated by a gap with heat carrier, in the case of steady conditions, contact of the fuel layer with the casing of the malfunctioning fuel assembly occurs at the initial moment. The diagram and initial state of the system are shown in Fig. 1. Suppose that the fuel layer is static, with adiabatic conditions on the side opposite the casing wall. Assuming no lateral and axial heat transfer, this system may be regarded as one-dimensional. It is obvious that satisfaction of this condition entails sufficient extent of the fuel layer along the wall.

Heat transfer inside each of the three theoretical regions of the given physical approximation is described by nonsteady heat-conduction Eq. (1), taking account of q_{VF} for the

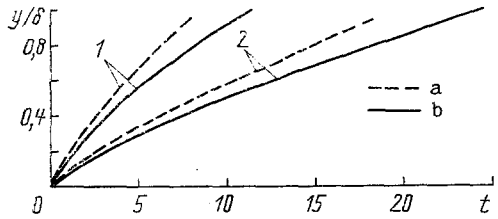

Fig. 2

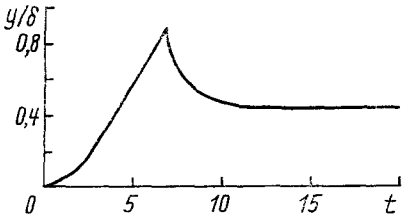

Fig. 3

Fig. 2. Comparison of the results of approximate analytical and numerical solution for a single wall with constant fluxes at the boundaries, with retention of the molten wall layer: 1) $q_{s2} = 0$; 2) $q_{s2} = 0.5\,q_{s1}$; a) numerical solution; b) approximate analytical solution.

Fig. 3. Motion of the phase interface in the wall of the emergency casing of a fuel assembly from contact with the molten fuel to melting and the subsequent establishment of the steady state.

layer of heat-liberating fuel. The heat transfer from the fuel to the casing wall is described by a boundary condition of the fourth kind [1]. At the boundary of the casing wall of the adjacent fuel assembly with the circulating heat carrier, the condition of convective heat transfer is assumed. The heat conduction in the gap between the casing walls is calculated as the heat conduction of the heat carrier enclosed there and, after evaporation of the latter, as the heat conduction of its vapor. At the beginning of phase transition, the condition in Eq. (2) at the phase boundary must be considered.

The system of three differential equations with conditions at the boundaries is solved numerically by the finite-difference method. The finite-difference scheme for the heat-conduction boundary problem with discontinuous coefficients may be obtained by an integrointerpolational method [2, 3]. An absolutely stable purely implicit scheme is used here [3]

$$\frac{1}{\tau^{j+1}} c_i^{j+1} (T_i^{j+1} - T_i^j) =$$

$$= \frac{1}{\hbar_i^{j+1} h_i^{j+1}} [\lambda_i^{j+1} (T_{i-1}^{j+1} - T_i^{j+1}) - \lambda_{i+1}^{j+1} (T_i^{j+1} - T_{i+1}^{j+1})] + q_V, \tag{7}$$

where

$$T_i^{j+1} = T(x_i,\ t_{j+1});\ \hbar_i^{j+1} = (h_i^{j+1} + h_{i+1}^{j+1})/2;$$

$$\lambda_i^{j+1} = \lambda(x_{i-1/2},\ t_{j+1});\quad c_i^{j+1} = c(x_i,\ t_{j+1});$$

$$q_V = \begin{cases} q_{VF} & \text{in the fuel layer,} \\ 0 & \text{in the casing walls.} \end{cases}$$

To retain the order of accuracy when a moving discontinuity appears, a nonuniform spatial grid is used; this grid is chosen at each time step so that one of its points coincides with the point of discontinuity, the phase interface y(t), i.e., one point of the grid is floating and moves with the phase boundary until it approaches a specified extreme position. The time step is taken to be variable, varying when the displacement of the phase boundary in the given step τ^{j+1} moves toward the given maximum or minimum value. In problems with phase transitions, a scheme with a variable time step and a nonuniform spatial grid with floating points allows the required accuracy to be obtained with fewer spatial grid points than in schemes using a uniform spatial grid, such as a scheme with a variable time step and displacement of the phase boundary by one spatial grid point at a time or a scheme with a fixed time step and fractional spatial steps in determining the position of the phase interface [4].

The conditions at the boundaries of the regions are written in the form

$$T_1^{j+1} = \varkappa_1 T_2^{j+1} + \mu_1,\ T_N^{j+1} = \varkappa_2 T_{N-1}^{j+1} + \mu_2, \tag{8}$$

where

$$\varkappa_1 = 1;\ \mu_1 = \frac{h_2^{j+1}}{\lambda_2^{j+1}} q_{s1}^{j+1};$$

$$\varkappa_2 = \frac{1}{1+B}\,;\ \mu_2 = \frac{B}{1+B}\,T^{j+1}_{e,i0}\,;$$

$$B = \frac{h^{j+1}_N/\lambda^{j+1}_N}{h^{j+1}_{e,2}/\lambda^{j+1}_{e,2}},\ i0 = 2 \quad \text{in the case of ideal contact with the neighboring region (with subscript e);}$$

$$B = (h^{j+1}_N/\lambda^{j+1}_N)\,\alpha_2,\ i0 = 1 \quad \text{in the absence of ideal contact.}$$

The matching condition at the phase boundary in Eq. (2) is written in the form

$$T^{j+1}_{Ny2} = \varkappa_m T^{j+1}_{Ny2+1} + \mu_m, \tag{9}$$

where

$$\varkappa_m = 1,\ \mu_m = \frac{h^{j+1}_{Ny2+1}}{\lambda^{j+1}_{Ny2+1}}\left[\lambda^{j}_{Ny1}\,\frac{T^{j}_{Ny1-1} - T^{j}_{Ny1}}{h^{j}_{Ny1}} - \frac{R_m(y^{j+1} - y^{j})}{\tau^{j+1}}\right].$$

A combination of fitting and iterative methods is used to solve the nonlinear system in Eqs. (7)-(9) [3]. At each time step, the fitting procedure is repeated several times, until the required accuracy of the temperature distribution obtained in the system is achieved. The next displacement of the phase boundary Δy^{j+1}, as in one of the schemes of [4], is fractional relative to the spatial grid steps, and is taken to be Δy^{j} in the first iteration (ky = 1). If after the fitting procedure the difference between the temperature at the point on the phase boundary $T^{j+1,ky}_{Ny2}$ and the melting point T_M falls outside the required accuracy, then $\Delta y^{j+1,ky+1}$ in the next iteration is calculated from the expression

$$\Delta y^{j+1,ky+1} = \frac{\tau^{j+1}}{R_m}\left(\lambda^{j}_{Ny1}\,\frac{T^{j}_{Ny1-1} - T^{j}_{Ny1}}{h^{j}_{Ny1}} - \lambda^{j+1,ky}_{Ny2+1}\,\frac{T_m - T^{j+1,ky}_{Ny2+1}}{h^{j+1,ky}_{Ny2+1}}\right). \tag{10}$$

This algorithm for numerical solution of the conjugate Stefan problem is realized in the MDUCT2 computational program in FORTRAN-IV for an EC-type computer.

To test the quality of the numerical method, its results are compared with the results of analytical solution of Eq. (5). In the numerical solution, ten points of the spatial grid are taken, and then the algorithm (with squeezing of the spatial grid at the external boundary) allows the calculation to be continued up to the melting of 93% of the wall thickness. Table 1 gives the results for the particular case when the approximate analytical solution of Eq. (5) is accurate: with complete removal of the molten wall material (b = 1) and zero heat extraction from the external surface (q_{S2} = 0). The other parameters of the problem are as follows: q_{S1} = 156 W/cm^2, R_M = 2100 J/cm^3, a_L = 0.018 cm^2/sec, a_S = 0.055 cm^2/sec; this corresponds approximately to the conditions of reactors with sodium coolant [5]. It is evident from Table 1 that the numerical solution coincides with the analytical solution to two significant figures. In Fig. 2, curves of y(t) are shown for two values of the flux from the surface of the solid wall layer (q_{S2} = 0 and q_{S2} = 0.5 q_{S1}) in the case of complete retention of the molten layer. It is evident that the approximate analytical solution gives a somewhat lower melting rate of the wall. The difference in values of y(t) here is no more than 20%.

The MDUCT2 program is used for calculations of the transient thermal interaction of molten fuel with the steel casing walls of the malfunctioning and neighboring fuel assemblies for the initial state in Fig. 1. The parameters required for the calculations are taken from [5]. In Fig. 3, results are given for the case of retention of the molten wall material in the contact zone. It follows from the results obtained that melting of the casing wall of the malfunctioning fuel assembly begins at initial contact with the molten fuel. After 1.3 sec, the heat carrier boils up, filling the gap between the walls. The casing wall of the malfunctioning fuel assembly melts more rapidly on account of the deterioration in heat transfer through the gap, and is completely molten 6.8 sec after the initial contact. Then it is assumed in the model that, after complete melting of the first wall, direct contact sets in between the molten material and the casing wall of the neighboring fuel assembly. As a result, the wall material of the casing wall of the malfunctioning fuel assembly again solidifies over more than half of the wall thickness. The temperature distribution becomes

steady; the casing wall of the malfunctioning fuel assembly is only partially in the molten state here, and the casing wall of the neighboring fuel assembly remains undamaged. Calculation for the case of runoff of molten wall material from the contact zone shows that the internal wall melts somewhat more rapidly: in 4.6 sec from the moment of contact. However, in both cases, if normal heat extraction from the casing wall of the neighboring fuel assembly is maintained, this wall remains undamaged.

NOTATION

x, coordinate; t, time; T, temperature; λ, thermal conductivity; c, specific heat of unit volume; a, thermal diffusivity; R_m, latent heat of fusion; T_m, melting point; δ, thickness of casing wall; X_1, coordinate of the molten-layer boundary; y, coordinate of the phase boundary; q_{s1}, heat flux to the internal (left-hand) boundary; q_{s2}, heat flux from the external (right-hand) boundary; α_2, heat-transfer coefficient from the external boundary; t_s, time of melting of wall; q_{VF}, heat liberation in the fuel layer; τ, time step; h, spatial step of the grid. Indices: ℓ, liquid phase; s, solid phase; i, number of the spatial grid point; j, number of the time step; N_{y1}, N_{y2}, numbers of the spatial grid points at the phase interface in the j-th and (j + 1)-th steps, respectively; N, number of spatial grid points; ky, number of iteration.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Moscow (1967).
2. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Moscow (1977).
3. A. A. Samarskii, Theory of Difference Schemes [in Russian], Moscow (1983).
4. L. I. Rubinshtein, The Stefan Problem [in Russian], Riga (1967).
5. G. B. Usynin, A. S. Karabasov, and V. A. Chirkov, Optimizational Models of Fast Reactors [in Russian], Moscow (1981).

TWO-MODE MODEL OF FLOW IN A PLASMOTRON CHANNEL

N. F. Aleshin and A. F. Bublievskii

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The characteristics of an electric arc in a turbulent gas flow are calculated on the basis of the concept of laminar flow in the arc zone.

The methods of calculating electrical arcs in a cylindrical channel which are known in the literature are usually based on the assumption that the flow conditions, which depend on the parameters of the external gas flow blown through the arc, are the same (either laminar or turbulent) over the whole channel cross section. At the same time, taking account of the specific properties of the electric arc allows the flow in the plasmotron channel to be represented in the form of central laminar flow and outer turbulent flow in many cases.

Estimates for various gases show that, at moderate Re (up to 10^5), calculated from the input parameters, and at sufficiently high temperatures in the central region of the flow ($\sim$15,000 K or more), the mean turbulent thermal conductivity over the channel cross section is approximately an order of magnitude lower than the molecular thermal conductivity, while the corresponding viscosity values are comparable with one another. Similar results were obtained in [1], where it was indicated that the heat transfer in the axial of a plasmotron channel may be regarded as laminar.

Note also the possibility of decrease in the temperature pulsations in a plasma arc on account of radiant heat transfer between turbulent eddies and rapid "deexcitation" of the highest-temperature formations [2]. The result is that turbulent heat transfer is negligibly small, and there is practically no pulsational component of the heat flux for optically dense media with very intense radiation.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.

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